S.G.V.C VidyaPrasarak Trust's

M.G.V.C ARTS, COMMERCE AND SCIENCE COLLEGE MUDDEBIHAL

DEPARTMENT OF MATHEMATICS

Teachers Use ICT enabled tools for Effective teaching learning process

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Topic- COMPLEX NUMBERS

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1 Complex Numbers

De• nitions

De• nition 1.1 *Complex numbers are de• ned as ordered pairs* (x, y)

Points on a complex plane. Real axis, imaginary axis, purely imaginary numbers. Real and imaginary parts of complex number. Equality of two complex numbers.

De• nition 1.2 *The sum and product of two complex numbers are de• ned as follows:*

$$\begin{aligned}(x_1, y_1) + (x_2, y_2) &= (x_1 + x_2, y_1 + y_2) \\ (x_1, y_1) \cdot (x_2, y_2) &= (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)\end{aligned}$$

In the rest of the chapter use $z, z_1, z_2, \ldots$ for complex numbers and x, y for real numbers. introduce i and $z = x + iy$ notation.

Algebraic Properties

1. Commutativity
$$z_1 + z_2 = z_2 + z_1, \qquad z_1 z_2 = z_2 z_1$$
2. Associativity
$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3), \qquad (z_1 z_2) z_3 = z_1 (z_2 z_3)$$
3. Distributive Law
$$z(z_1 + z_2) = z z_1 + z z_2$$
4. Additive and Multiplicative Indentity
$$z + 0 = z, \qquad z \cdot 1 = z$$
5. Additive and Multiplicative Inverse
$$\begin{aligned}-z &= (-x, -y) \\ z^{-1} &= \left(\frac{x}{x^2 + y^2}, \frac{-y}{x^2 + y^2}\right), \qquad z \neq 0\end{aligned}$$

6. Subtraction and Division

$$z_1 - z_2 = z_1 + (-z_2), \qquad \frac{z_1}{z_2} = z_1 z_2^{-1}$$

7. Modulus or Absolute Value

$$|z| = \sqrt{x^2 + y^2}$$

8. Conjugates and properties

$$\begin{aligned}
\overline{z} &= x - iy = (x, -y) \\
\overline{z_1 \pm z_2} &= \overline{z_1} \pm \overline{z_2} \\
\overline{z_1 z_2} &= \overline{z_1}\,\overline{z_2} \\
\overline{\left(\frac{z_1}{z_2}\right)} &= \frac{\overline{z_1}}{\overline{z_2}}
\end{aligned}$$

9.

$$\begin{aligned}
|z|^2 &= z\overline{z} \\
\operatorname{Re} z &= \frac{z + \overline{z}}{2}, \operatorname{Im} z = \frac{z - \overline{z}}{2i}
\end{aligned}$$

10. Triangle Inequality

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

Polar Coordinates and Euler Formula

1. Polar Form: for $z \neq 0$,

$$z = r(\cos\theta + i\sin\theta)$$

where $r = |z|$ and $\tan\theta = y/x$. θ is called the argument of z. Since $\theta + 2n\pi$ is also an argument of z,the principle value of argument of z is take such that $-\pi < \theta \leq \pi$. For $z = 0$ the arg z is unde• ned.

2. Euler formula: Symbollically,

$$e^{i\theta} = (\cos\theta + i\sin\theta)$$

3.

$$\begin{aligned}
z_1 z_2 &= r_1 r_2 e^{i(\theta_1 + \theta_2)} \\
\frac{z_1}{z_2} &= \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)} \\
z^n &= r^n e^{n\theta}
\end{aligned}$$

4. de Moivre's Formula

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

Roots of Complex Numbers

Let $z = re^{i\theta}$ then

$$z^{1/n} = r^{1/n}\exp\left(i\left(\frac{\theta}{n}+\frac{2k\pi}{n}\right)\right)$$

There are only n distinct roots which can be given by $k = 0, 1, \ldots, n-1$. If θ is a principle value of $\arg z$ then θ/n is called the principle root.

Example 1.1 *The three possible roots of* $\left(\frac{1+i}{\sqrt{2}}\right)^{1/3} = \left(e^{i\pi/4}\right)^{1/3}$ *are* $e^{i\pi/12}, e^{i\pi/12+i2\pi/3}, e^{i\pi/12+i4\pi/3}$.

Regions in Complex Plane

1. $\epsilon - nbd$ of z_0 is de• ned as a set of all points z which satisfy
$$|z - z_0| < \epsilon$$
2. *Deleted nbd* of z_0 is a nbd of z_0 excluding point z_0.
3. Interior Point, Exterior Point, Boundary Point, Open set and closed set.
4. Domain, Region, Bounded sets, Limit Points.